Circuit Based Battery Models: A Review

Francisco M. González-Longatt

Abstract—**This paper is a general review of several circuit-based battery model in the bibliography. The paper does not supply detailed information on how to identify the several parameters of the models, and only define a short family of models, but not discuss the exhaustive use of these models. Two main families of models are shown, elementary and dynamic. A general explication of the dynamic model is presented.**

Index terms— **Batteries, modeling.**

I. INTRODUCTION

Electrochemical batteries are of great importance in electrical power systems because they give a means for storing energy in a way that is immediately available. Some of the main battery uses that have grown fast during last decade are [1]:

- Batteries within Uninterruptable Power Supplies (UPS).
- Battery Energy Storage Plants (BESP) to be installed in power grids with the purpose of compensating active and reactive powers (in this sense they are an extension of the SVCs, and therefore are sometimes called also SWVCs).
- Batteries of the main energy source of electric vehicles.

There are many types of batteries that are currently being used - or being developed for use, the more widespread batteries are the lead–acid ones, in the two main kinds of flooded and Valve-regulated types.

An extensive research has been carried out to develop new types of batteries and converters to convert the batteries output into useful work [2] there exist some models developed by experts of chemistry [1]–[5], they are too complex for a practical application of electrical engineer; in addition, they are not expressed in terms of electrical networks, that would help know-how in the analyzes. Still exists a noticeable lack of battery models, expressed in a way manageable, and phenomenological.

This paper provides a non exhaustive summary of the battery models. And the conclusion of this paper results an compressive and simple summarize of the most important and relevant: elementary and dynamic models.

II. BATTERY PARAMETERS

There are several parameters associated with battery modeling [6], and these parameters are briefly explained below:

1. Internal Resistance
   - Self-discharge Resistance: its which takes account of resistances in (a) electrolysis of water at high voltage and (b) slow leakage across the battery terminal at low voltage. This resistance is more temperature sensitive and inversely proportional to the temperature change [6].
   - Resistances for Charge and Discharge (R_c/R_d): These are the resistances associated with electrolyte resistance, plates resistance and fluid resistance, however all these resistances can be different in charging and discharging.
   - Overcharge and Over-discharge Resistance: When the battery is overcharged or over-discharged, the internal resistance will be increased significantly due to the electrolyte diffusion. These resistances are attributed largely to the electrolyte diffusion during over charging and over discharging.
2. Discharge Type:
   - Continuous Discharging: When battery continuously delivers energy to load without rest, and the battery capacity is dropping continuously.
   - Intermittent Discharging: When a battery drives **a** load for a period and is disconnected from the load for some time, then voltage recovery will be took place in the battery to increase its voltage with some amount. When the battery is operating in this intermittent manner, it will give a longer discharge time.
3. Discharge Mode:
   - Constant Load: When a battery delivers energy to a load of constant resistance, so the load current is decreasing as battery voltage does.
   - Constant Current: Current drawn from a battery is kept constant to a load that continuously reduces its resistance, the discharge duration in this mode is shorter due to the average current is higher. The voltage drops faster than that in constant load.
   - Constant Power: A constant electrical power is drawn by load from a battery, such that the load current will be increasing to compensate for the decreasing battery voltage. This mode has the

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F.G.L is on Universidad Nacional Experimental Politécnica, Carretera Nacional Maracay-Mariara, Departamento de Ingeniería Eléctrica, Maracay, Estado Aragua, Venezuela, Tlf. +58-243-5546954, Fax: +58-243-554642, E-mail: fglongatt@iee.org.

He is PhD. student at Universidad Central de Venezuela, Los Chaguaramos, Facultad de Ingeniería, Escuela de Ingeniería Eléctrica, Caracas, Distrito Capital, Venezuela, Tlf. +58-414-4572832, E-mail: flongatt@elecrisc.ing.ucv.ve.

shortest discharge time.

4. Rate of Charge/Discharge: To extend the service life of battery the rate of charge and discharge can not be too high. Excessive overcharging and over-discharging can reduce battery life. Further, the frequency of switching needs to be taken into account.

III. ELEMENTARY BATTERY MODELS

There are many models available for battery modeling, and a few of them are mentioned here in this section. The merits and demerits of those models are given.

A. *Simple Battery Model*

The most commonly used battery model is shown in Figure 1. This model consists of an ideal battery with open-circuit voltage E_o and a constant equivalent internal series resistance ESR (R_{int}). V_o is the terminal voltage of battery.

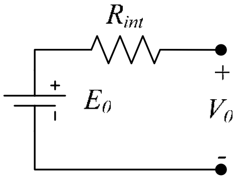


Fig. 1. Simple Battery Model

V_o can be obtained from the open circuit measurement and ESR can be obtained from both the open circuit measurement and one extra measurement with load connected at the terminal when the battery is fully charged. While this model has been extensively used, it does not take into account the varying characteristic of the internal impedance of the battery with the varying state of charge, electrolyte concentration and sulfate formation. Such a model is only applicable in some circuit simulations where the energy drawn out of the battery is assumed to be unlimited or where the state of charge is of little importance [2]

B. *Modified Model*

Jean Paul Cun [9] proposed an improved battery model based on the configuration given in Figure 1. In this battery model, the battery's state of the charge is taken into account, by making the ESR of battery no longer constant, but varies in accordance with its state of charge. A common formula is to set:

$$ESR = \frac{R_0}{S^k} \tag{1}$$

where R_0 = initial battery internal resistance calculated when the battery is full charged and:

$$S = 1 - \frac{Ah}{C_{10}} \tag{2}$$

Where C_{10} is the ten-hour capacity (Ah) at the reference temperature (this value varies as the battery ages). S varies from 0 (battery discharged) to 1 (battery charged). k is a coefficient that is a function of the discharge rate, calculated on the basis of k_1, k_2, and k_3. k_1. k_2 and k_3 are coefficients determined using the curves provided by the manufacturers. They correspond to three discharge rates. This model has been used by many battery manufacturers for battery monitoring purposes.

C. *Thevenin Battery Model [10]*

The other commonly used model is the Thevenin battery model, which consists of an ideal no-load battery voltage (E_0), internal resistance (R), capacitance (CO) and overvoltage resistance (R_0). CO represents the capacitance of the parallel plates and R_0 represents the non-linear resistance contributed by the contact resistance of plate to electrolyte (Fig. 2)

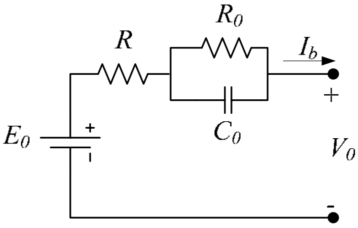
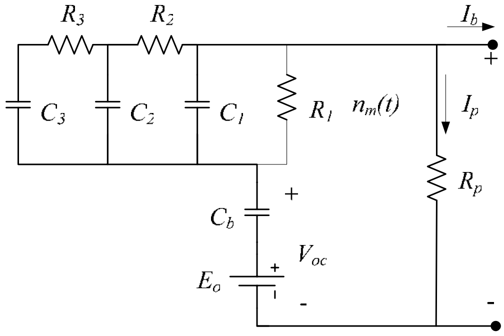


Fig. 2. Thevenin Battery Model

The main disadvantage of the Thevenin battery model is that

all the elements are assumed to be constant, but in fact all the values are functions of battery conditions. An improvement upon the Thevenin models is a lineal electrical battery model, shown in Fig. 3 [11]. This model [12] uses linear components o account for self-discharge (R_p) and various over-voltages ($n_m(t)$ network). Thought more accurate, this model however does not take in account temperature dependence and uses different sets of elements values to model the battery at different states of charge. Thus a continue battery evaluation becomes tedious [12].



Fig, 3. Linear Electrical Model

D. *Resistive Thevenin battery model*

The resistive Thevenin battery model [7] is similar to the model described in last section, but it assumes the following:

- The electrodes are made of porous materials.
- The electrolytic resistance is constant throughout discharge.
- The discharge occurs at constant current.
- Polarization is a linear function of the active material current density.

The circuit diagram for resistive Thevenin battery model is shown in Figure 5.

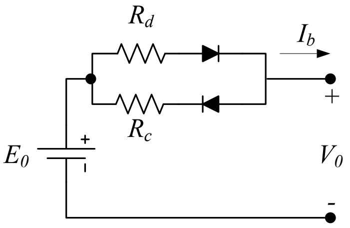


Fig. 5. Thevenin Battery Model

This model has two kinds of internal resistances, R_c and R_d, which are associated with the charging and discharging process of the battery, respectively. These two parameters (R_c and R_d) model all forms of energy loss which includes electrical and non-electrical losses. The diodes, shown in Figure 5, implies that the during charging or discharging only one of the resistances R_c or R_d (which is in series with the forward biased diode) will be used because when one diode is forward biased the other will be reverse biased. These diodes are present only for modeling purposes only and have no physical significance in the battery. While this model is better than the previous model described in last section, it does not account for the capacitance effect such as the transient current conditions occurring in the battery.

E. Modified Thevenin equivalent battery model

A simple dynamic battery model presented in [8] was considered by Farrell in [7] and this model is shown in Figure 6. The dynamic equations of the circuit model for discharging and charging are given by:

$$\begin{aligned} \frac{dV_p}{dt} &= -V_p \frac{1}{R_d C} + V_0 \frac{1}{R_d C} - I_b \frac{1}{C}, \quad V_p \le V_0 \\ \frac{dV_p}{dt} &= -V_p \frac{1}{R_c C} + V_0 \frac{1}{R_c C} - I_b \frac{1}{C}. \quad V_p > V_0 \end{aligned} \tag{1}$$

Where:

$$I_b = \frac{V_p - V_0}{R_b} \tag{2}$$

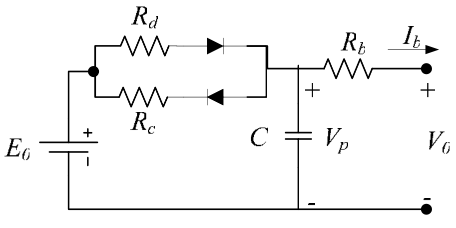


Fig. 6. Modified Thevenin Equivalent Model

The direction of battery discharging current is considered to have positive sign throughout this paper. In the special case when R_b = 0, then V_p = V_0, and the battery current is determined solely by the load resistance. The capacitance C represented in Fig. 4, is the polarization capacitance.

F. Model [8]

The model proposed by [8] is developed from a series experimental test were performed through examination of the graphic plots of the experimental data, and manufacturers specifications.

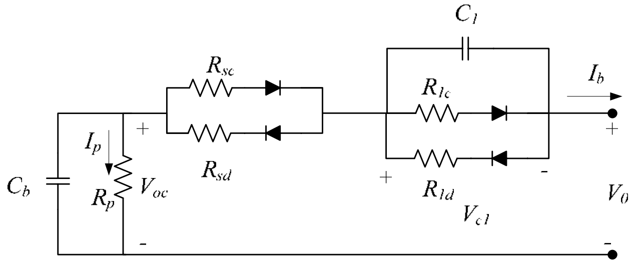


Fig. 7. Lineal Model by [11]

Ideal diodes, chosen strictly for directional purposes, were required to differentiate between internal and overvoltage resistances for charge and discharge.

G. Dynamical Model

An empirical mathematical model is developed in [7,8] to model lead-acid traction battery:

$$V_0 = V_{oc} - \left(R_b + \frac{K}{SOC} \right) I_b \tag{3}$$

Where K = polarization constant, typically 0.1 ohm, SOC = state of charge. The improvement of this model is to account for the non-linear characteristic of both the open circuit voltage and internal resistance represented by the K/SOC component.

IV. DYNAMIC MODEL

A dynamic model, that consider a real interaction with external word, include environmental temperature (θ_a), and battery current as input (I_0), and battery voltage (V_0) as well as the extracted charge (Q_e)and electrolyte temperature as outputs (θ) (Fig. 8).

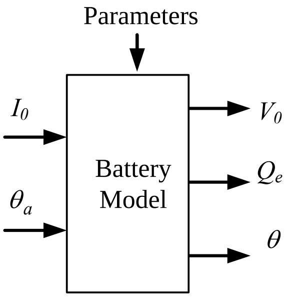


Fig. 8. Generic Dynamic Model of a Battery

This model of a battery can be represented by an equivalent electric network reported in Fig. 9. The resistances and capacitances shown are function of the battery state-of-charge and electrolyte temperature. This model has mainly two important parts: main (reversible) reaction branch, and the parasitic reaction branch. The main branch, have a number n of R-C block, that simulate the dynamic behavior of the battery, this circuit are not constant, since they depend on the battery stat-of-charge and electrolyte temperature (with good degree of approximation, however, the quantities, $\tau_k = R_k C_k$, can be kept constant). On the other hand, the parasitic reaction branch simulates the notable current drawn only during the charge process (and at the end of it).

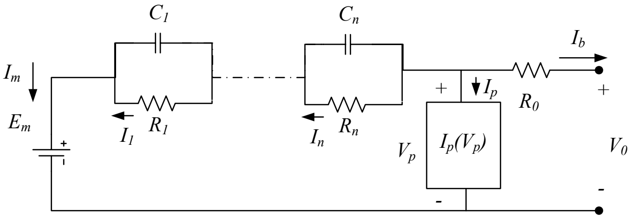


Fig. 9. Dynamic Battery Model [13]

The battery modeling requires the identification of several circuit elements, under different state of charge. At fixed discharge current *I,* positive when exiting the battery, the dependence of the capacity in the electrolyte temperature θ (expressed in °C and supposed constant) can be expressed with a good approximation by:

$$C(I,\theta)_{I,\theta=const} = C_0(I)\left(1+\frac{\theta}{-\theta_f}\right)^{\varepsilon} \qquad (\theta>\theta_f)$$

Where: θ_f is the electrolyte freezing temperature that depends mainly of the electrolyte specific gravity (normally be assumed -40°C). $C_0(I)$, is a empirical function of discharge current, and obviously, equal to the battery capacity at 0°C; from experimental results this quantity can be expressed of a reference current I^*.

$$C_0(I)=\frac{K_c C_0^*}{1+(K_c-1)\left(\frac{I}{I^*}\right)^{\delta}}$$

$C_0^* = C_0(I^*)$. K_c and δ are an empirical coefficients, constant for a given battery and a given I^*.
Finally the capacity of the battery at electrolyte temperature and discharge current constant:

$$C_0(I,\theta)=\frac{K_c C_0^*\left(1+\frac{\theta}{-\theta_f}\right)^{\varepsilon}}{1+(K_c-1)\left(\frac{I}{I^*}\right)^{\delta}} \tag{4}$$

During transients the constant hypothesis has been experimentally confirmed, still valid, the real battery current *i(t)*,a filtered value of this current I_{avg}, is used. Good results are obtained taking $I_{avg} = I_k$, where I_k is the current flowing in one of the resistor R_k (the actual *k* depends on the particular model consider). To quantify the level of discharge of the battery, two different numbers are sued:
State-of –charge:

$$SOC = 1-\frac{Q_e}{C(0,\theta)} \tag{5}$$

Deep of charge:

$$DOC = 1-\frac{Q_e}{C(I_{avg},\theta)} \tag{6}$$

Where $Q_e(t)$ is given:

$$Q_e = \int_0^t -I_m(\tau)d\tau \tag{7}$$

The dynamic equation that allows the electrolyte temperature computation is, simply:

$$C_\theta \frac{d\theta}{dt} = \frac{\theta-\theta_e}{R_\theta} + P_s \tag{8}$$

A. Third-Order Model

This model is constituted by:

- An electrical equivalent with two *R-C* blocks and an algebraic parasitic branch (Fig. 10).
- Algorithms for calculate the state of charge and internal (electrolyte) temperature.
- Equations for computations of the elements of the equivalent network as functions of state of charge and temperature.

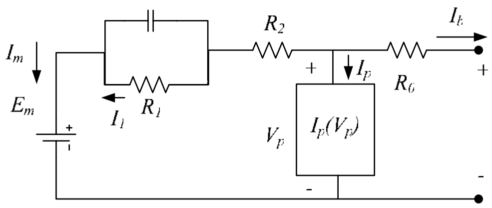


Fig. 10. Third-Order Battery Model

The assumed state of variables is the currents I_1 and I_2, and the state of charge Q_e, and the electrolyte temperature. The dynamic equations of the models are therefore:

$$\begin{aligned}
\frac{dI_1}{dt} &= \frac{1}{\tau_1}(I_m - I_1)\\
\frac{dQ_e}{dt} &= -I_m\\
\frac{d\theta}{dt} &= \frac{1}{C_\theta}\left[P_s - \frac{(\theta-\theta_a)}{R_\theta}\right]
\end{aligned} \tag{9}$$

Where $\tau_1 = R_1C_1$, and:

$$\begin{aligned}
E_m &= E_{m0} - K_E(273+\theta)(1-SOC)\\
R_0 &= R_{00}\left[1+A_0(1-SOC)\right]\\
R_1 &= -R_{10}\ln(DOC)\\
R_2 &= R_{20}\frac{e^{\left[A_{21}(1-SOC)\right]}}{1+e^{\left(\frac{A_{22}I_m}{I^*}\right)}}
\end{aligned} \tag{10}$$

Where E_{m0}, K_E, R_{00}, A_0, R_{10}, R_{20}, A_{21}, A_{22} are constant for a particular battery. *DOC* and *SOC* are as defined in equations (5), (6), and the current to be utilized in the expression of the capacity $C(I,\theta)$ = is $I_{avg} = I_1$.

The behavior of the parasitic branch is actually strongly non-linear. The expression of I_p as function of V_p, that matches the Tafel gassing-current relationship is:

$$I_p = V_p G_{p0} e^{\left(\frac{Vp}{V_{p0}}+A_p\left(1-\frac{\theta}{\theta_f}\right)\right)} \tag{11}$$

This expression contains the parameters G_{p0}, V_{p0}, A_p, that are constant for a particular battery.

V. CONCLUSIONS

- The recent increase in the use of batteries in power systems makes necessary reliable models for design and simulation of the batteries, especially lead-acid.

- In this paper, circuit-based models for batteries were presented, from the simple and complicated dynamic models. A family of models adapted to different levels of precision, was presented, and the contribution of this paper is the summary of models.

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