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Demonstration of an Inertia Constant Estimation Method Through Simulation

Peter Wall
Peter.Wall@postgrad.manchester.ac.uk

Francisco González-Longatt
Senior Member, IEEE
fglongatt@ieee.org

Vladimir Terzija
Senior Member, IEEE
terzija@ieee.org

***Abstract*-The inertia constant of a system describes the initial, transient, frequency behaviour of that system when subjected to a real power disturbance. Therefore, the inertia constant of a system can be a useful tool when investigating the frequency stability of a system. The use of the swing equation is a viable method for estimating the inertia constant, if a measurement system that can provide time stamped measurements of the frequency and power dynamics during a disturbance is available. An example of such a system is a Wide Area Monitoring system that is capable of monitoring the frequency at all, or a select set, of generation sites. A method for estimating the inertia constant based on this data is developed and demonstrated for two simple networks. This demonstration is performed using computer simulations in the DigSILENT PowerFactory software package. The inertia constant estimation method is implemented in the MATLAB environment.**

***Index Terms*-- Electric power generation, frequency measurement, power system measurement, power system transients, inertia constant, swing equation, electro-mechanical transient processes.**

I. Introduction

As the liberalisation of the power industry develops, increasing priority has been placed upon commercial drivers for the operation of a power system [1]. This trend has led to a number of new developments in the operation of power systems, an example of which is security limits being relaxed in order to maximise the exploitation of installed assets.

Replacement of existing preventative control with real-time corrective control would allow improvements to be made in the exploitation of available network assets. This improvement would be possible because corrective actions would be based on the actual system state, rather than the system state anticipated at the planning stage, allowing more efficient control actions and a more relaxed set of security constraints [1].

However, the introduction of corrective control is heavily dependent upon improving the access operators would have to information regarding the real-time state and stability of the network. One form of stability is the systems ability to maintain the frequency of supply at approximately the nominal frequency, according to the standardized requirements and the given statutory and operational limits.

The system frequency is dependent upon the real power balance within that system. Any change in this power balance, whether caused by a change in load or generation, will result in an instantaneous change in the system frequency. The initial change in frequency caused by any such load-generation imbalance is due to a change in speed, acceleration or deceleration, of the various rotating masses in the system [2], [3]. This change in speed occurs because the rotating masses attempt to adapt to the new load-generation balance by using the kinetic energy stored within them to accommodate any temporary imbalance. In the case of an excess of generation (equivalent to a load decrease), the excess energy will be stored in the rotating masses causing them to accelerate, and the system frequency to increase. In the case of a deficit of generation (load increase or generator outage), the power not supplied by the system generation will be drawn from the rotating masses causing them to decelerate, and the system frequency to decrease.

As the relationship between the initial frequency behaviour after a power imbalance is determined by the behaviour of a system's rotating masses, the bulk of which is provide by generators, it is possible to describe it mathematically using a property of the generators in the system [2], [3].

This property is the inertia constant, which is defined as the time taken, in seconds, for a generator to replace its stored kinetic energy when operating at rated speed and apparent power output. It links the rate of change of frequency in the instant after a real power disturbance to the magnitude of that disturbance through the swing equation. It is possible to estimate the inertia constant of a system, or individual generator, using this equation if measurements of the frequency and power balance during a real power disturbance are available.

A Wide Area Monitoring Systems (WAMS), of the sort currently being used and developed around the world [4], could provide the necessary measurements [5].

The inertia constant of a system is likely to become an increasingly dynamic property in the future. This is because the available generation will move away from the traditional portfolio, dominated by large thermal units, and become more diverse, with increasing use of intermittent generation and technologies with *low* or *zero inertia*. Furthermore, the introduction of other new technologies such as energy storage and smart grid devices will potentially affect the behaviour of the system frequency and power balance [1].

The increasingly dynamic nature of the inertia constant will mean that the frequency response of a system, to any given disturbance, will become less predictable. This is an issue as the frequency stability for a particular set of operating limits may be satisfactory when the system inertia is high but may become unsatisfactory if the inertia were to fall.

Therefore, the ability to estimate the system inertia constant may become attractive as it could allow system operators to accommodate variations in the inertia constant when making decisions. This may allow improvements in not only the security of a system but also the economic performance of a system, as the information gathered could, for example, allow an insight into the frequency support services a system operator should purchase, which can be a significant expense [5].

Previous work dealing with this application of the swing equation has produced estimates of a systems inertia constant for a variety of purposes. These include investigations of the available spinning reserve [7]; the nature of the relationship between the inertia constant of a system and the magnitude of the system load [8] and finally the inertia of wind turbines [9]. This work has produced beneficial results but there is no assessment of the influence that the nature/size of a disturbance or the properties of the network have upon the quality of the estimates produced.

The work presented in this paper demonstrates the use of the swing equation for inertia constant estimation and some of the factors that can affect the estimates produced. Section II describes how the swing equation is used to generate an estimate of the inertia constant. Section III describes the simulations performed to demonstrate the method and presents some analysis of the results of these simulations.

II. Inertia Constant Estimation Procedure

The method presented here is based on the swing equation and can be used to estimate the inertia constant of either an individual generator or an entire multi-machine system. The only difference between the two applications is that for an individual generator the frequency and power measurements used are taken from the terminals of that generator. Whilst, for the multi-machine case the *frequency of the inertia centre* and *net system power imbalance* are used. The definition and calculation of these two system properties are given in Section II.C.

A. Swing Equation

The swing equation defines the relationship between the real power balance Δp_i (p.u.) (between mechanical power p_{mi} and electrical power p_{ei}) and the rate of change of frequency df_i/dt (Hz/s), at generator i with an inertia constant of H_i (s) for the time immediately after a real power disturbance has occurred. It is assumed here that any damping effects are negligible in the time immediately after the disturbance. One frequently used form of the swing equation, which is valid immediately after a disturbance, is as follows:

$$\frac{2H_i}{f_n}\frac{df_i}{dt} = p_{mi} - p_{ei} = \Delta p_i \quad i = 1, 2, \ldots, N \tag{1}$$

The time immediately after the disturbance is defined as t=0$^+$, the relationship described by the swing equation is only valid for this time because after it has passed other factors, not accounted for here (Generation unit primary controls, loads' response, series compensation, storage, spinning reserve, HVDC, AGC, LFC etc), begin to influence the system frequency.

B. Application to an Individual Generator

The swing equation (1) can be used directly to make an estimate for the inertia constant of an individual generator, provided that reliable measurements of the frequency first derivative (rate of change of frequency) and power are available. In the simulations performed in the paper, the variables recorded are noise free, to eliminate noise as a cause of any variation seen in this preliminary demonstration.

The value of the derivative of the frequency required in the equation is calculated from the discretely sampled frequency values, in the following way. The difference between two adjacent frequency samples, in Hz, is divided by the time difference between when the two samples were taken. The value obtained is then assumed to be constant for the time period between the two frequency samples it is based on. This means that the derivative of frequency value treated as being recorded at t=0$^+$ is actually a sort of mean of the derivative of frequency for the period between $t = t_1$ and $t = t_1 + t_s$, where t_s is the period between the samples and t_1 is some arbitrary time for which a frequency sample exists.

The power imbalance at the generator is calculated directly from the difference between the electrical and mechanical powers sampled during the simulation and converted to a p.u. value on the system load base. These two values can then be used in a rearranged form of (1) to provide an estimate of the inertia constant of the generator.

C. Application to a Multi-machine System

In order to estimate the inertia constant of an entire system it is necessary to calculate a frequency representing the equivalent frequency of the system. This must be done because during large power imbalances the local frequencies of individual generators may not be the same.

In this paper the system frequency was calculated from the frequency of each individual generator based on the concept of the frequency of inertia centre. This concept is developed using the same reasoning that is used in mechanics to introduce the concept of a centre of mass. This frequency is referred to as f_c and corresponds to the inertia weighted average of all generator frequencies:

$$f_c = \frac{\sum_{i=1}^{N} H_i f_i}{\sum_{i=1}^{N} H_{ii}} = \frac{1}{H_T}\sum_{i=1}^{N} H_i f_i \tag{2}$$

where H_T is the total inertia of the system. By taking (2) into account the sum of the swing equations (1) for a system with N generators yields an expression for the dynamics of the frequency of the inertia centre:

$$\frac{2H_T}{f_n}\frac{df_c}{dt} = \Delta p = \sum_{i=1}^{N} \Delta p_i \tag{3}$$

where the variable Δp, corresponds to the net active power imbalance of the system and is calculated as the sum of the power imbalance at all in service generators on the system. It is necessary to convert this imbalance onto the same base as the system load. Once these two values are calculated they can be used in a rearranged form of (3) to give an estimate for the total inertia constant of the entire system:

$$H_T = \frac{f_n \sum_{i=1}^{N} \Delta p_i}{2\frac{df_c}{dt}} \tag{4}$$

The results of the simulations performed here are mostly presented in terms of the error in the estimate produced. This error is calculated using the following equation:

$$error = \left(\frac{H_t - H_e}{H_t}\right)100\% \tag{5}$$

where H_t is the true value of the inertia constant, of the generator or system, and H_e is the estimated value.

III. Single Bus Simulations

In this section the above method for estimation of inertia constant is implemented for the case of two different test networks. These networks were two single bus systems, one with a single generator (G) and a 100 MW load and another with three generators (G1, G2, G3) and a single load. In all cases, the generator model used was based on a 210 MVA, 50 Hz, gas turbine synchronous generator. These simulations were intended to demonstrate the results of applying the inertia constant estimation method and how the network and simulation approach can influence these estimates.

In most of the figures presented in this paper their will be two data series, one labelled **NI** and the other labelled **I**. The first data series (NI) is based on data with no interpolation and the second is based on data for which interpolation is used (I). This refers to two different ways in which the simulation package treats the time steps at which power and frequency data is generated. The first approach, with no interpolation, only generates a data point at the specified sample time interval. In this case, this is 0.02 s as one sample per cycle is a reasonable sample time for collecting phasor data [5]. The values taken for the power imbalance and derivative of frequency are from the second sample after the disturbance has occurred.

The second approach, were interpolation is used, is different because the simulation package will generate several additional points for the time after a disturbance; this is to improve the representation of transient behaviour in this period. These points are generated, based on interpolation between the original samples either side of the disturbance. This improved representation allows the values taken for the power imbalance and derivative of frequency to be taken from the first point after the disturbance has occurred.

A. *Estimation for a Step Load Change*

This set of simulations consisted of applying a step change in the real power drawn by the single static load of a simple single generator test system with a generator model based on a 210 MVA, 50 Hz, gas turbine synchronous generator with the inertia constant set to 7.334s on a 210 MVA base. The step changes considered took a range of -100% to 100%, of the real power, with an increment of 1%; the error in the inertia constant estimation for every disturbance, for both interpolated and non interpolated data, are shown in Fig. 1.

Comparison of the two data series in Fig. 1 reveals four interesting features. The first of these is that the estimation process is reliable; 96% of the estimate errors lie within the range of 1% to -1% for the non-interpolated data whilst for the interpolated data 93% of the estimate errors lie within the range of 0% to -2%.

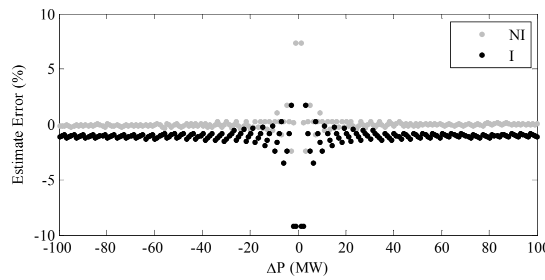

Fig. 1. *H* estimate errors for both interpolated (I) and non-interpolated (NI) power and frequency data for load changes of between -100 and 100%.

This difference between the ranges in which it is likely any inertia constant estimate, for the simulations performed here, will be found highlights the second feature in this plot. This is the clear difference of approximately -1% between the errors in the estimates performed using the non-interpolated and interpolated data. The plots presented in Fig. 2 show that the reason for this difference in the estimates is a difference in the derivative of frequency value returned by each method; the power imbalance (Δp) data is the same for both methods. This difference in the derivative is due to the interpolated data being generated for an earlier time than the non-interpolated data but both data sets have the same frequency value.

The third feature seen in Fig. 1 is that the errors seen increase exponentially for step changes with a magnitude of less than approximately 20%. This is because, for disturbances of this size, the derivative of frequency value recorded is only a few tenths and therefore any error in its value will become more significant. This is consistent with the slightly larger divergence seen for interpolated data than for non-interpolated data, 93% of points in a 2% range compared to 96%, and the small error in the interpolated derivative of frequency values seen in Fig. 2.

The final feature seen in Fig. 1 is the non-linear steps seen in the variation of the estimate error as the disturbance size falls. Fig. 3 shows these steps more clearly. These steps are caused by the small difference (1%) between each load step change that is simulated. The simulation of the frequency behaviour does not recognise these small changes and therefore several different load change simulations will have the same derivative of frequency value.

This value will initially be an underestimate of the true derivative of frequency value, but as the disturbance size is reduced, it will become an overestimate. This behaviour causes the estimate error to move toward 0%, or -1% in the interpolated case, as it becomes less of an underestimate and then pass through and away from this value as it becomes an overestimate. The non-linear steps then occur when the simulation package moves to a new derivative of frequency value, which will once more be an underestimate.

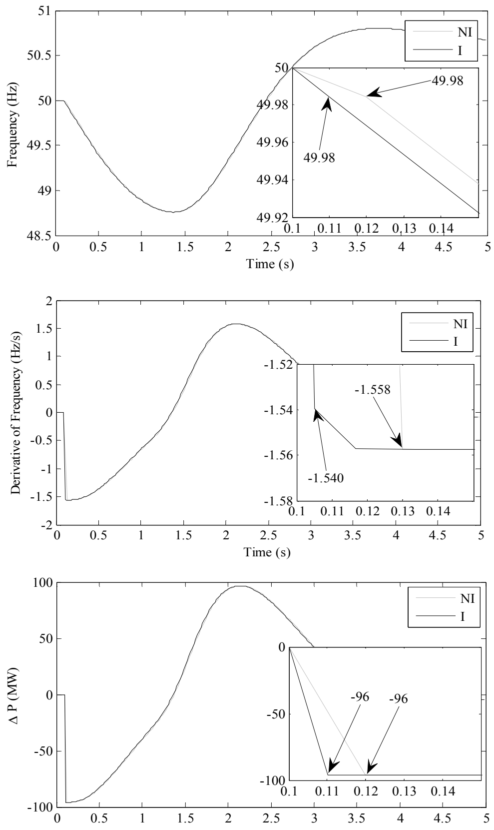

Fig. 2. Plots of frequency, derivative of frequency and real power imbalance for a load increase of 96%. Insets show a focused view of the data at t=0^+.

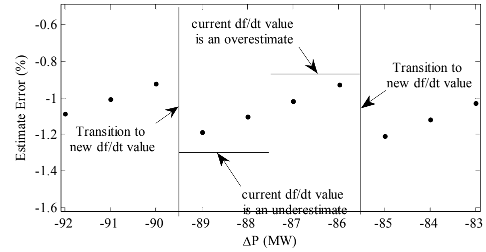

Fig. 3. Plot of the non-linear steps in the estimate errors with reference to the derivative of frequency value (*df/dt*) for clarity only interpolated data is used.

B. Influence of System Size

The rest of the single bus system simulations performed here use a one bus system with three gas turbine generators, G1, G2 and G3, of the type used in Section III.A (210 MVA, 50 Hz, H_t = 7.334 s) and one load, which is three times the size (300 MW). This simulation uses the same set of percentage step changes in the real power drawn by the load to demonstrate that the size of the system in terms of the installed capacity (in MVA) does not affect the estimation process. The inertia constant estimates for all three generators, for both interpolated and non-interpolated data, are shown in Fig. 4.

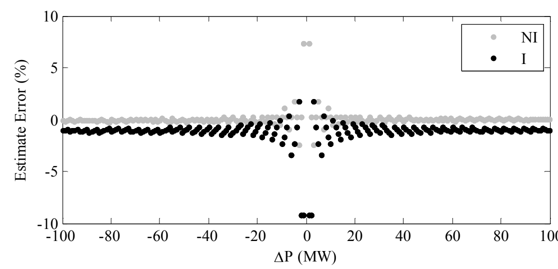

Fig. 4. *H* estimate errors for both interpolated and non-interpolated power and frequency data for load changes of between –100 and 100% in a one bus three generator system. The estimates for each generator are the same.

The estimates for each generator were identical to one another, and to those seen in Fig. 1, this is unsurprising given that the three generators are identical to one another, connected to the same bus and individually are responding to the same magnitude of power step as in Section III.A. However, this is still a useful result as it demonstrates that the method is independent of the installed capacity of a system so any variation in the methods behaviour seen for the other simulations performed for this system is due to other factors.

C. Generator Outage Disturbance

The inclusion of multiple generators allows the response of the estimation method to data from a generator disconnection disturbance to be demonstrated.

The simulations performed for generator disconnections consisted of increasing the initial load in the system from 50MW to 500MW, in steps of 5MW, and then disconnecting generator G3 from the system for each initial load level. It is important to note that, in this set simulations, when the real power was increased the reactive power was also increased to maintain the power factor of the load. In addition, the

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dispatch of each generator was adjusted so that they were always equal, in terms of both real and reactive power.

Estimates of the inertia constant of the disconnected generator (G3) can be produced, if the measurement system used is capable of continuing to produce measurement of frequency and power during the generators disconnection.

The error in the inertia constant estimates for these simulations are shown in Fig. 5,. Only the interpolated data is shown as it had the same characteristics as the non-interpolated data, apart from the –1% offset, and presenting both made the figure unclear. The estimates for G1 and G2 are slightly different from one another, which is contrary to the result seen in Section III.B, where the estimates for each generator are identical.

To determine if the difference in the estimates was due to the disturbance being a generator disconnection, rather than a step load change, the set of disconnection simulations were repeated with only real power being drawn by the load.

The estimate results for these simulations with interpolated data are shown in Fig. 6 and comparison of the estimates for G1 and G2 will show that they are the same. This would suggest that the differences seen in Fig. 5 are due to the reactive power drawn by the load rather than the fact that the disturbance is a generator disconnection. This is because in the first case, where the load had a reactive component, the disconnection of G3 involved both a real and reactive power disturbances unlike previous simulations which had only a real power disturbance.

This can be confirmed with reference to the data in Fig. 7, which shows that there is a small difference in the values of both the power imbalance and derivative of frequency data at $t=0^+$ for the case where reactive power is included in the load.

However, this difference is very small and causes a mean change in the estimates for G1 and G2 of 0.0394% and –0.0367% respectively. The very small change in error that reactive power causes means that investigation of its effects can be neglected from any future work.

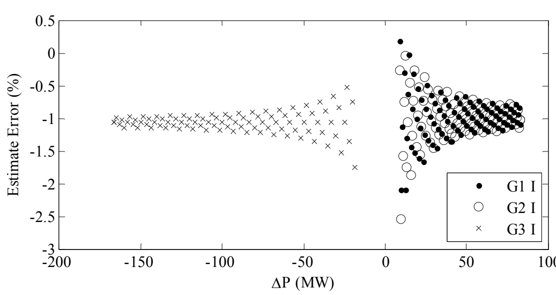


Fig. 5. Inertia constant estimate errors for generators G1, G2 and G3, for only interpolated data, when G3 is disconnected for a range of loads with power factor of 0.9487.

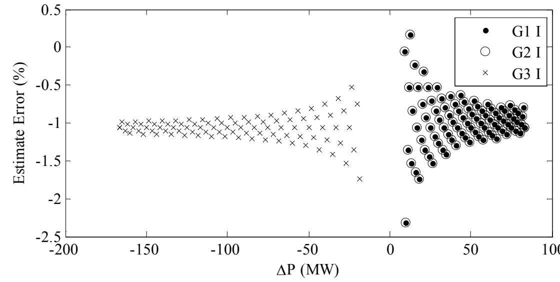
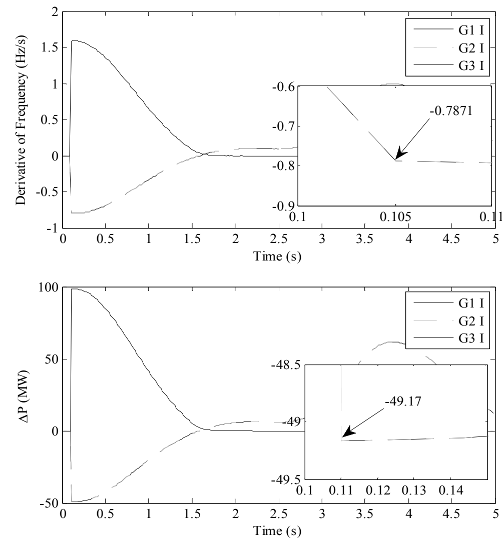
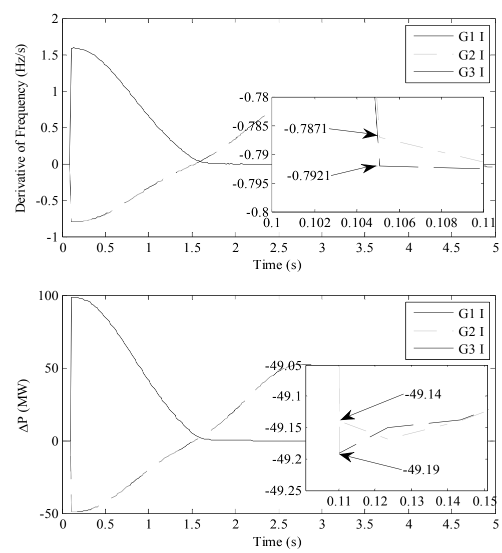


Fig. 6. Inertia constant estimate errors for generators G1, G2 and G3, for only interpolated data, when G3 is disconnected for a range of loads with power factor of 1.



(a)



(b)

Fig. 7. Plots of derivative of frequency and power imbalance for disconnection of G3 for loads with real power consumption of 300MW for a power factor of 1 (a) and 0.9487 (b). Insets show a focused view of $t=0^+$.

P. Wall, F. Gonzalez-Longatt, and V. Terzija, "Demonstration of an inertia constant estimation method through simulation," in *Universities Power Engineering Conference (UPEC)*, 2010 45th International, 2010, pp. 1-6.

D. *Influence of Generator Inertia Constants*

In order to demonstrate the influence of a generators inertia constant on the estimation method two sets of simulations were performed. The first of these involved setting the inertia constant of the three generators, G1, G2 and G3, to 3.667s, 7.334 and 14.668s respectively and repeating the set of load changes seen in Section III.B. The second set of simulations also repeated the set of load changes used in Section III.B but with the inertia constants of all three machines set to 3.667s, half of the original inertia constant. The estimates obtained for these simulations are presented in Fig. 8 and Fig. 9 respectively. The insets in these figures show that when non-interpolated (NI) data is used the errors in the estimates of the inertia constant of each machine vary in the first case but not in the second.

This demonstrates that the reliability of the estimation method is not dependent on the absolute value of the inertia constant. However, their does seem to be some dependence on the value of an individual machines inertia constant relative to the inertia constant of the system. This relationship is quite clear in the case of estimates based on non-interpolated data, but less so in the case of estimates based on interpolated data.

The trend seen in the non-interpolated results is that for machines with low inertia constants (G1), relative to the system, overestimates of the inertia constant will occur, whilst the opposite is true for machines with high inertia constants (G3), relative to the systems inertia constant. The generator with the higher inertia constant (G3) causes this change in the errors by suppressing the frequency response of the generator with the lower inertia constant (G1) by increasing the size of its own frequency response. This interaction causes G3 to have a larger derivative of frequency value for the power imbalance and therefore, from (1), the estimate of the inertia constant will be an underestimate and vice versa for G1. The impact of this interaction is only clear in the estimates based on non-interpolated data because of the delay in collecting the data, compared to the interpolated data, allows the effects of this interaction time to develop.

The size of this variation in the estimates for the non-interpolated data is probably larger than would be seen in a real system due to the unrealistic case of three different generators being located at the same bus, but any significant sensitivity of an estimation method to the parameter being estimated is a potential issue.

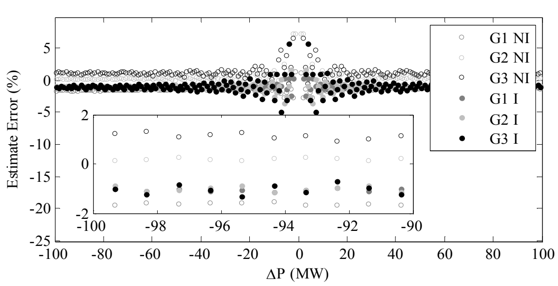


Fig. 8. Estimate errors for a system with three generators G1, G2 and G3 with inertia constants of 3.667, 7.334 and 14.668 respectively. Inset shows focused view of errors to clearly show variation in error with relative values.

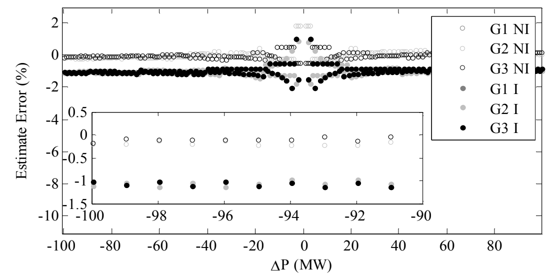


Fig. 9. Estimate errors for a system with three generators G1, G2 and G3 with equal inertia constants of 3.667s. Inset shows focused view of errors to clearly show no variation in error with absolute value.

IV. Conclusions

In this paper a new method for estimation of the inertia constant of power systems is presented. It is based on the use of the generator swing equation and measurement of the system frequency and its rate of change, as well as knowledge of how large a power imbalance has occurred. The results of very thorough simulation and testing have demonstrated that, in the networks considered in the paper, the proposed inertia constant estimation method produces very reliable estimates of the inertia constant. This conclusion is based on over 90% of all the estimates generated having errors in the ranges of 1% to -1% and 0% to -2% for estimates based on non-interpolated and interpolated data respectively.

The potential error in any estimates produced using this method does diverge away from the likely ranges, described, in cases where a small derivative of frequency value is encountered. Such small values were encountered in 10% of the simulations performed; however, this divergence only occurs for disturbances that are very small relative to the system itself and as such do not present a danger to the system frequency stability. The non-linear steps present in the estimate errors are a function of how the software used handles small numbers rather than the estimation method.

The difference in the likely range of any error is one of the main differences between the use of the interpolated and non-interpolated data generated by the simulation package. The fixed error of approximately -1% in the estimates based on interpolated date is undesirable but in future, more complex, work this error is likely to be preferable to the dependence upon the relative inertia constants of a systems generators that is present in estimates based on non-interpolated data.

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