Paper accepted for presentation at the 2011 IEEE Trondheim PowerTech

Unscented Kalman Filter for Frequency and Amplitude Estimation

Happy Novanda, Pawel Regulski, Francisco M. González-Longatt, *Senior Member IEEE* and Vladimir Terzija, *Senior Member, IEEE*

***Abstract*—This paper introduces a new digital signal processing algorithm for frequency and amplitude estimation based on Unscented Kalman Filter (UKF). The results of computer simulated and realistic synthetic data tests are presented. The initial parameters used during the tests were chosen carefully using an established parameter estimation method, the Self Tuning Least Square (STLS). It is concluded that the proposed algorithm is simple, efficient and has low computational demands compare to STLS which makes the UKF a very promising method in next generation of power quality monitoring devices.**

***Index Terms*— Kalman filters, unscented transformation, power quality, frequency estimation, amplitude estimation.**

I. Introduction

THE development of numerical algorithms for estimating power quality (PQ) parameters has become an important subject in recent years. More reliable methods are required for power quality monitoring and estimation. In general, power system voltage and current waveforms are distorted by harmonic and interharmonic components, particularly during system disturbance. Faults or other switching transients may change the magnitude and phase angles of the waveforms. Moreover, voltage and current can also be distorted by non-linear loads, power electronic components and inherent non-linear nature of the system elements [1]. The assessment of PQ can be done either by calculating, measuring or estimating PQ indices (frequency, spectrum, harmonic distortion etc.). The estimation of PQ indices is still an important and yet challenging research area to be explored.

In the past different techniques have been developed for PQ indices estimation. Fast Fourier Transformation (FFT) [2], Least Squares (LS) [3-4], Newton Type Algorithm (NTA) [5-7] and Self Tuning Least Square (STLS) [8-10] are some of acknowledged signal processing methods used in the frequency and amplitude measurement.

One of the most popular methods for solving nonlinear parameter estimation problems is Extended Kalman Filter (EKF) [11-13]. The EKF linearizes nonlinear systems so that Kalman Filter equations can be applied. In practice the EKF has several disadvantages such as when the assumptions of local linearity are violated, linearization can produce highly unstable filters. This might lead to divergence phenomena [11].

Another drawback of the EKF is that the use of Jacobian matrices to linearize the nonlinear model equations often leads to significant implementation difficulties, particularly increased processing requirements and execution time, making the algorithm difficult for real-time applications [11].

More iterative methods based on Kalman filter have been proposed in [16, 17]. This approach can overcome accuracy problems of the EKF due to neglecting non-linear terms. However, the iteration procedure leads to higher processing time.

This paper carries out a study of *Unscented Kalman Filter* (UKF). The UKF is a novel measuring technique based on Unscented Transformation (UT) theory. This method was originally proposed in [11], [15]. The UKF method can eliminate the disadvantages of the EKF because it does not linearize the non linear systems. Instead, it uses a statistical distribution of the state which is propagated through the nonlinear equations. This approach provides better estimates of the actual state and the posterior covariance matrix [14, 15].

In this paper, the computer simulated and realistic synthetic data records are utilized to analyze the validity and performance of the proposed UKF algorithm for estimation of frequency and amplitude of the processed signal (arbitrary voltage, or current).

II. Methodology

A. Unscented Transformation

The *Unscented Transformation* (UT) is a transformation based on the insight that it is easier to approximate a Gaussian distribution than a highly nonlinear function [15]. The idea of UT is to deterministically choose a set so called *sigma points* with mean $\bar{x}$ and covariance $\mathbf{P_{xx}}$ and find the mean $\bar{y}$ and covariance $\mathbf{P_{yy}}$ by propagating sigma points through a nonlinear transformation. In UT, the $2n+1$ sigma points can be obtained by formula below:

$$\chi_0 = \bar{x} \tag{1}$$

$$\chi_i = \bar{x} + \left(\sqrt{(n+\lambda)\mathbf{P}_{xx}}\right)_i \tag{2}$$

$$\chi_{i+n} = \bar{x} - \left(\sqrt{(n+\lambda)\mathbf{P}_{xx}}\right)_i \tag{3}$$

H. Novanda (rrhappy.novanda@postgrad.manchester.ac.uk) and P. Regulski (pawel.regulski@postgrad.manchester.ac.uk) are PhD students at the School of Electrical and Electronic Engineering, The University of Manchester, Manchester, UK.

F. González-Longatt (e-mail: fglongatt@ieee.org) is Postdoctoral Research Associate at the same University.

V. Terzija (e-mail: terzija@ieee.org) is the EPSRC Chair Professor at the same University.

978-1-4244-8417-1/11/$26.00 ©2011

where $\left(\sqrt{(\mathrm{n}+\lambda)\mathbf{P}_{xx}}\right)_{\mathrm{i}}$ is the i*th* column of matrix $\sqrt{(\mathrm{n}+\lambda)\mathbf{P}_{xx}}$ and $\lambda=\alpha^2(\mathrm{n}+\kappa)-\mathrm{n}$. Parameter α is suggested to be between 10^{-4} and 1.0 [15] and value of parameter κ is $3-n$ or 0.

After carefully selected, sigma points are then propagated through the following function:

$$\boldsymbol{\gamma}_{\mathrm{i}}=f(\boldsymbol{\chi}_{\mathrm{i}}),\ \text{where } i=0,1,...,2n \tag{4}$$

The next step is calculating mean and covariance of the propagated points given by:

$$\bar{\mathrm{y}}=\sum_{\mathrm{i}=0}^{2\mathrm{n}}\mathrm{W}_{\mathrm{i}}^{\mathrm{m}}\boldsymbol{\gamma}_{\mathrm{i}} \tag{5}$$

$$\mathrm{P}_{yy}=\sum_{i=0}^{2n}W_i^c[(\gamma_i-\bar{\mathrm{y}})(\gamma_i-\bar{\mathrm{y}})^T] \tag{6}$$

The weights W_i^m and W_i^c are defined below:

$$W_0^{\mathrm{m}}=\frac{\lambda}{\mathrm{n}+\lambda} \tag{7}$$

$$W_0^{\mathrm{c}}=\frac{\lambda}{(\mathrm{n}+\lambda)}+(1-\alpha^2+\beta) \tag{8}$$

$$W_i^m=W_i^c=\frac{1}{2(\mathrm{n}+\lambda)} \tag{9}$$

B. *Unscented Kalman Filter*

UKF is an algorithm which can solve nonlinear systems in the following form:

$$\mathbf{x}_{k+1}=\mathrm{f}(\mathbf{x}_k)+\mathbf{q}_k \tag{10}$$

$$\mathbf{y}_{k+1}=\mathrm{h}(\mathbf{x}_{k+1})+\mathbf{r}_{k+1} \tag{11}$$

where **x** is a discrete state vector, **y** is discrete measurement vector, **q** and **r** are the system and measurement Gaussian noises with zero mean and covariance matrices **Q** and **R,** respectively.

There are three stages in the UKF method [14, 15]:

1. Sigma points calculation

Firstly, we have to define an initial state vector $\mathbf{x}_0$, initial covariance $\mathbf{P}_0$, process noise covariance and measurement-noise covariance **Q** and **R**, respectively. This have to be defined in advance based on *a priori* knowledge of the system. In this paper, initial parameters are chosen carefully based on STLS algorithm results.

In this stage, sets of $2n+1$ sigma points are created based on previous state with following formula:

$$\mathbf{X}_{\mathrm{k-1}}=[\mathbf{x}_{\mathrm{k-1}}\ \cdots\ \mathbf{x}_{\mathrm{k-1}}]+\sqrt{(\mathrm{n}+\lambda)}[\mathbf{0}\quad\sqrt{\mathbf{P}_{\mathrm{k-1}}}\quad-\sqrt{\mathbf{P}_{\mathrm{k-1}}}] \tag{12}$$

2. Kalman filter state prediction

Then sigma points in the first stage are propagated through function below:

$$\boldsymbol{\chi}^*_{k|k-1}=\mathrm{f}(\boldsymbol{\chi}_{k-1}) \tag{13}$$

The following step is computing the predicted state mean vector $\bar{\mathbf{x}}_{k|k-1}$, and predicted covariance matrix $\mathbf{P}_{k|k-1}$:

$$\bar{\mathbf{x}}_{k|k-1}=\sum_{i=0}^{2\mathrm{n}}W_i^m\boldsymbol{\chi}^*_{i,k|k-1} \tag{14}$$

$$\mathbf{P}_{k|k-1}=\sum_{i=0}^{2\mathrm{n}}W_i^c[(\boldsymbol{\chi}^*_{i,k|k-1}-\bar{\mathbf{x}}_{k|k-1})(\boldsymbol{\chi}^*_{i,k|k-1}-\bar{\mathbf{x}}_{k|k-1})^T]+\mathbf{Q} \tag{15}$$

3. Kalman Filter state correction

Next, the sigma points related to the predicted state mean vector and covariance matrix are calculated with the following formula:

$$\boldsymbol{\chi}_{k|k-1}=[\bar{\mathbf{x}}_{k|k-1},\bar{\mathbf{x}}_{k|k-1}\pm\sqrt{(\mathrm{n}+\lambda)\mathbf{P}_{k|k-1}}] \tag{16}$$

Then, the sigma points are propagated through measurement-update function:

$$\boldsymbol{\gamma}_{k|k-1}=\mathrm{h}(\boldsymbol{\chi}_{k|k-1}) \tag{17}$$

The following step is calculating propagated points:

$$\bar{\mathbf{y}}_{k|k-1}=\sum_{i=0}^{2\mathrm{n}}W_i^m\boldsymbol{\gamma}_{i,k|k-1} \tag{18}$$

Then, we can obtain the measurement covariance matrix $\mathbf{P}_{yy}$ and cross-covariance of the state and measurement $\mathbf{P}_{xy}$:

$$\mathbf{P}_{yy}=\sum_{i=0}^{2\mathrm{n}}W_i^c[(\boldsymbol{\gamma}_{i,k|k-1}-\bar{\mathbf{y}}_{k|k-1})(\boldsymbol{\gamma}_{i,k|k-1}-\bar{\mathbf{y}}_{k|k-1})^T]+\mathbf{R} \tag{19}$$

$$\mathbf{P}_{xy}=\sum_{i=0}^{2\mathrm{n}}W_i^c[(\boldsymbol{\chi}_{i,k|k-1}-\bar{\mathbf{x}}_{k|k-1})(\boldsymbol{\gamma}_{i,k|k-1}-\bar{\mathbf{y}}_{k|k-1})^T] \tag{20}$$

Finally, we can compute Kalman gain, the state mean and covariance below:

$$\mathbf{K}_k=\mathbf{P}_{xy}\mathbf{P}_{yy}^{-1} \tag{21}$$

$$\bar{\mathbf{x}}_k=\bar{\mathbf{x}}_{k|k-1}+\mathbf{K}_k(\mathbf{y}_k-\bar{\mathbf{y}}_{k|k-1}) \tag{22}$$

$$\mathbf{P}_k=\mathbf{P}_{k|k-1}-\mathbf{K}_k\mathbf{P}_{yy}\mathbf{K}_k^T \tag{23}$$

III. Algorithm Testing

The UKF algorithm has been designed in such a way that he following instantaneous parameter model of the processed input voltage (it can also be a current) was used:

$$u(t)=U\sin(\omega t+\varphi)+\xi(t) \tag{24}$$

in which $u(t)$ is an instantaneous voltage at time t, U is the magnitude of fundamental component, ω is fundamental angular velocity, equal to $2\pi f$ where f is frequency, φ is the phase angle and $\xi(t)$ is a zero mean random noise. In this model, the higher harmonics are considered as a random noise and need to be properly filtered out. In the testing of the algorithm this filtering has not been done, just to demonstrate the original algorithm properties without some extra measures, which might only improve the overall algorithm performance. The only unknown model parameters are the signal magnitude, its frequency and phase.

The above algorithm has been tested thoroughly using computer simulated data records. Three types of test were carried out: static and random noise tests, dynamic tests, and synthetic data test.

A. Static and Random Noise Test

During the static test, the following signal model was used:

$$u(t) = \cos(\omega t + 30) + 0.3\cos(3\omega t + 90) + 0.2\cos(5\omega t + 150) \quad (25)$$

where ω is fundamental angular velocity which is equal to $2\pi f$ where $f = 50$ Hz.

The signal model above was then used to investigate the sensitivity of the method to random noise and selection of the initial covariance. In this simulation, the sampling frequency used was $f_s = 1.6$k Hz. Fig. 1 presents the algorithm sensitivity to random additive noise without changing the initial covariance. It is shown that the algorithm can track amplitude and frequency with low maximum error under very noisy condition. However, these maximum errors can be reduced by changing the initial covariance as shown in Fig. 2.

The *Signal to Noise Ratio* (SNR) is defined below:

$$SNR = 20\log\left(\frac{S}{\left(\sqrt{2}\sigma\right)}\right) \quad (26)$$

where $\frac{S}{\sqrt{2}}$ is the root mean square value of the signal (S_{RMS}) and σ is standard deviation of the noise. The maximum errors are calculated using function below.

$$\max(error) = \frac{\max(f_{est} - f_{real})}{f_{real}} \quad \text{or} \quad \frac{\max(U_{est} - U_{real})}{U_{real}} \quad (27)$$

The maximum errors versus the SNR are plotted below.

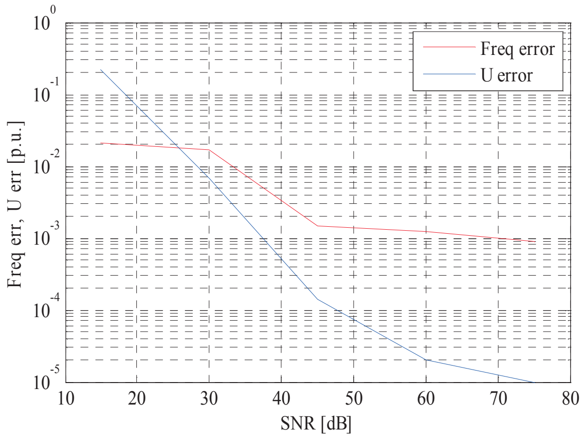

Fig. 1. Maximum amplitude and frequency errors vs. signal to noise ratio with constant initial covariance.

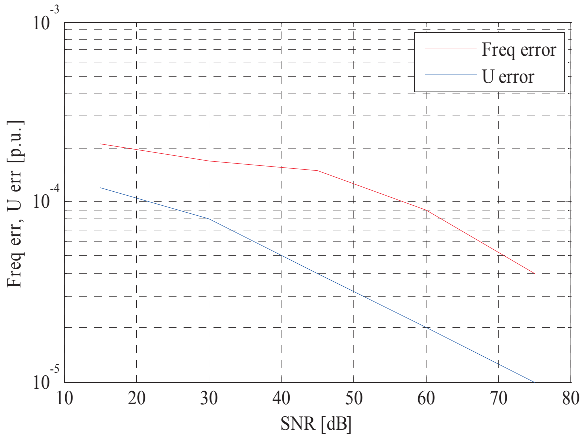

Fig. 2. Maximum amplitude and frequency errors vs. signal to noise ratio with adjusted initial covariance.

B. Dynamic Test

Dynamic tests were also carried out to analyze the algorithm sensitivity to signal and frequency distortion. The distorted voltage and current signals used for dynamic test were given below.

$$u(t) = \cos(\omega t + 45) + 0.5\cos(3\omega t + 120) + \ldots$$
$$\ldots + 0.3\cos(5\omega t + 150) + 0.2\cos(7\omega t + 280) \quad (28)$$

$$i(t) = 0.9\cos(\omega t + 45) + 0.4\cos(3\omega t + 60) + \ldots$$
$$\ldots + 0.2\cos(5\omega t + 30) + 0.1\cos(7\omega t + 130) \quad (29)$$

During the test, the signal parameters are dynamically changed. For the period t<0.158s, the test signals only consist of the fundamental component, which is the first term of equation (28), (29). At t=0.158s, the input signals were distorted with higher harmonics according to (28), (29) and frequency signal was changed significantly from f = 50 Hz to f=45 Hz and then linearly raised with the rate 1 Hz/s. In Fig. 3, input signals were presented.

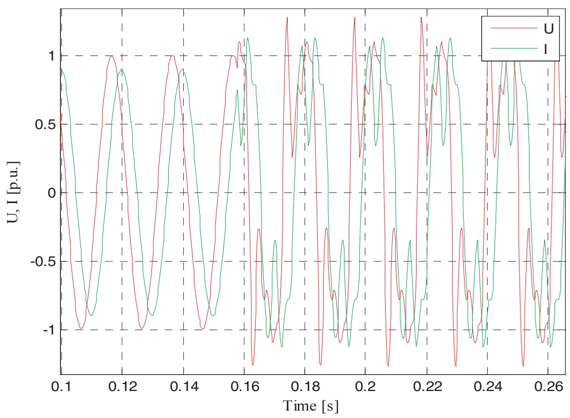

Fig. 3. Input signal for dynamic test.

It is shown in Fig. 4 that the UKF successfully tracked the amplitude. The highest error during this measurement excluding the convergence period is less than 10^{-2}%. In Fig. 5, the result of estimated frequency is compared to the real one. The algorithm convergence properties are determined by initial covariance. Faster convergence period can be obtained by reducing measurement noise covariance and vice versa.

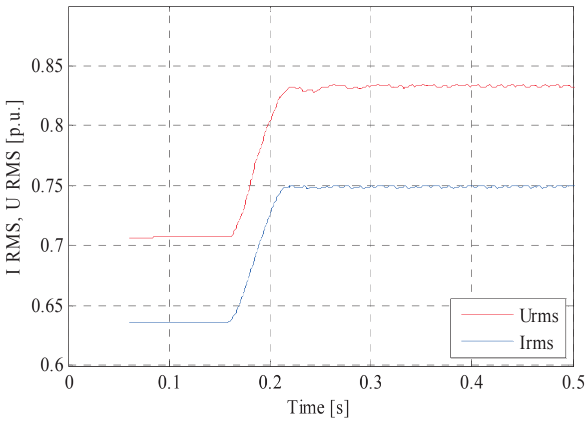

Fig. 4. Estimated RMS using UKF in a dynamic simulation.

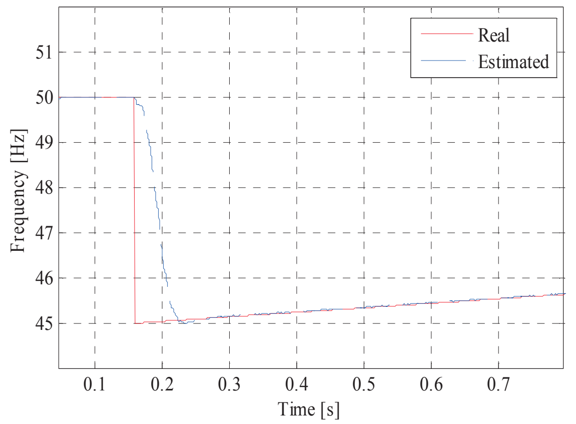

Fig. 5. Estimated frequency using UKF in a dynamic simulation.

C. Processing of a Steady State Distorted Signal Obtained Through Dynamic Simulation of a Power System

Fig. 1 shows the voltage waveforms of a three-phase transformer which is energized on a 500 kV network at $t = 0.05s$. The transformer rated 450 MVA, 500 kV/230 kV/60 kV consists of three windings connected in Y/Y/Delta. The voltage on phase “a” contains a high level of 4th harmonic, a details of the high contains of harmonic is shown in Fig. 2 and Fig. 8.

In Fig. 9, the estimated three-phase RMS voltages are presented. Before the disturbance, all three phase RMS voltages are around 3.07×10^{5}V. It is shown that the algorithm can track amplitude precisely under higher harmonics content. Note that by knowing estimated RMS voltage and current, power components can be calculated based on the definition given in the IEEE Standard 1459-2000 [22].

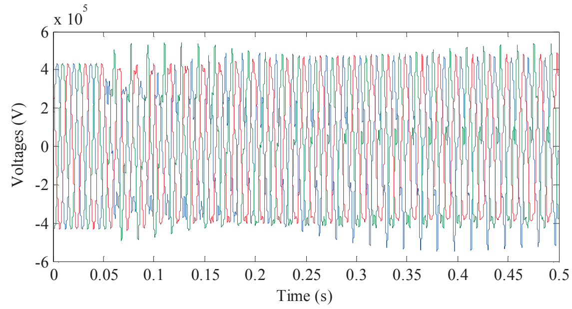

Fig. 1. Voltage waveforms data records.

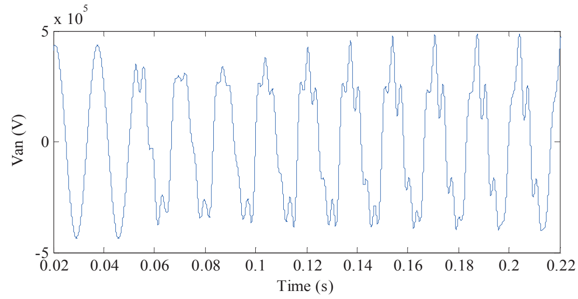

Fig. 2. Details of the voltage on phase *a*, transformer is energized at t = 0.05 s.

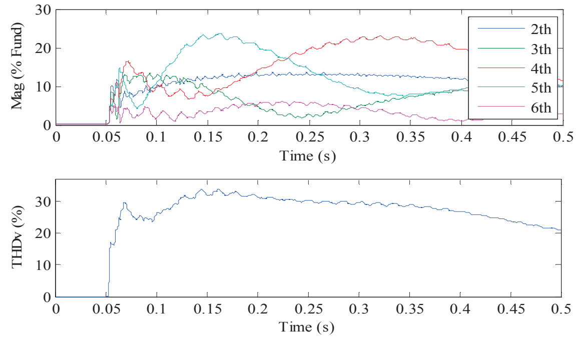

Fig. 3. Harmonic content on phase *a*.

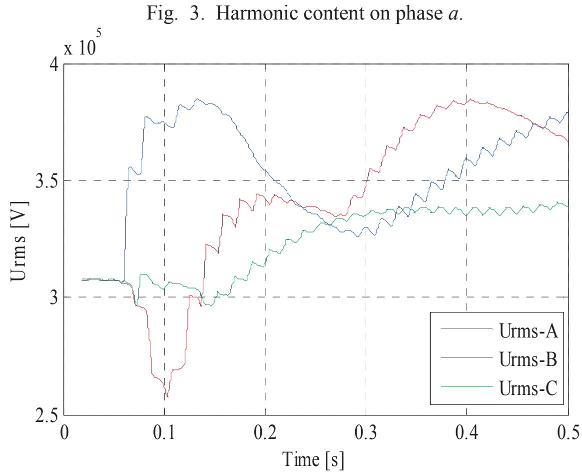

Fig. 9. Estimated three-phase RMS voltage

Fig. 10 reveals the system frequency before, during and after disturbance. The estimated result is compared with real frequency. The exact same value can be directly recognized from Fig. 10. This result verifies that UKF is capable to track frequency during large disturbance.

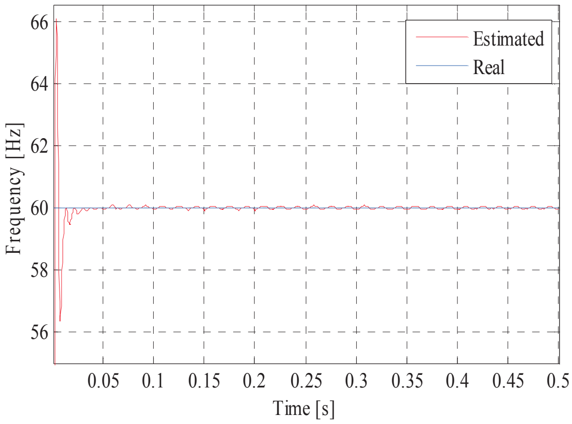

Fig. 10. Estimated frequency

IV. CONCLUSION

In this paper, a new parameter estimation method for frequency, amplitude and phase tracking, based on Unscented Kalman Filter is presented. Various simulations are carried out to analyze its steady state and dynamic performance. It is shown that UKF obtained high estimation accuracy both under normal and noisy conditions. It is verified that this method is not sensitive to frequency changes, thus it can be effectively implemented as a reliable tool for PQ indices estimation. Finally, the simplicity of the UKF due to the absence of the model linearization gives promising tracking performances and efficiency for PQ indices monitoring in harmonics polluted power systems. The authors are now extensively using the proposed algorithm for PQ indices estimation for data records obtained through measurements in real systems.

V. ACKNOWLEDGMENT

The authors wish to thank Prof. Knud Ole Helgesen Pedersen from Siemens Wind Power Denmark, Prof. Jacob Østergaard and Dr. John Eli Nielsen from DTU Denmark for the support to complete this paper. Also thanks to Schlumberger Foundation for the support through Faculty for The Future program.

VI. REFERENCES

[1] R.C.Dugan, M.F. McGranaghan, S. Santoso and H.W. Beaty, *Electrical Power Systems Quality,* 2nd. ed. McGraw-Hill, 2002.

[2] P.K. Dash, S.K. Panda, B. Mishra and D.P. Swain, "Fast estimation of voltage and current phasors in power networks using an adaptive neural network," *IEEE Trans. Power System*, vol. 12, no. 4, pp. 1494-1499, 1997.

[3] M.D.A. Rahaman and K.B. Yu, "Total least squares approach for frequency estimation using linear prediction," *IEEE Trans. On Acoustics, Speech and Signal Processing*, vol. 35, no. 10, pp. 1440-1454, 1987.

[4] R.Chudamani, K.Vasudevan and C.S.Ramalingam, "Real-Time Estimation of Power System Frequency Using Nonlinear Least Squares", *IEEE Trans. on Power Delivery*, vol. 24, issue: 3, pp.1021-1028, 2009.

[5] V. Terzija, M. Djuric and B. Kovacevic, "Voltage phasor and local system frequency estimation using newton type algorithm," *IEEE Trans. Power Delivery*, vol. 9, no. 3, pp. 1368-1374, 1994.

[6] V. Terzija, V. Stanojevic, M. Popov and L. van der Sluis, "Digital metering of power components according to IEEE standard 1459-2000 using the Newton-type algorithm," *IEEE Trans. Instrumentation and Measurement*, vol. 56, no. 6, 2007.

[7] V. Terzija, V. Stanojevic, M. Popov and L. van der Sluis, "Digital metering of power components according to IEEE standard 1459-2000 using the Newton-type algorithm," *IEEE Trans. Instrumentation and Measurement*, vol. 56, no. 6, 2007.

[8] V. Terzija, M. Djuric and B. Kovacevic, "A new self-tuning algorithm for the frequency estimation of distorted signals," *IEEE Trans. Power Delivery*, vol. 10, no. 4, 1995.

[9] V. Terzija and V. Stanojevic, "STLS algorithm for power-quality indices estimation," *IEEE Trans. Power Delivery*, vol. 23, no. 2, 2008.

[10] H. Novanda, V. Stanojevic and V. Terzija, "Power quality indices estimation during sudden generator disconnection in Faroe Islands power system," presented at International Conference on Advanced Power System Automation and Protection, Jeju, Korea, 2009.

[11] S. J. Julier and J. K. Uhlmann, "A New Extension of the Kalman Filter to Nonlinear Systems", in *Proc. of the 4th Conference on Signal Processing, Sensor Fusion and Target Recognition*, vol. 3068, pp. 182-193, 1997.

[12] B.F.L. Scala and R.R. Bitmead, "Design of an extended Kalman filter frequency tracker," *IEEE Trans. On Signal Processing*, vol.44,pp.739-742, 1996.

[13] P.K. Dash, R.K. Jena, G. Panda and A. Routray, "An extended complex Kalman filter for frequency measurement of distorted signals," *IEEE Trans. on Instrumentation and Measurement*, vol. 49, pp. 746-752, 2000.

[14] G. Valverde and V. Terzija, "Unscented kalman filter for power system dynamic state estimation," *IET Generation, Transmission & Distribution*, vol. 5, pp. 29-37, 2010.

[15] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," in *Proceedings of the IEEE*, vol. 92, pp. 401-422, 2004.

[16] N. Bretas, "An iterative dynamic state estimation and bad dataprocessing," *Int. Journal Electric Power Energy System*, vol. 11, no.1, pp. 70-74, 1989.

[17] E. Blood, B. Krogh and M. Ilic, "Electric power system static state estimation through Kalman filtering and load forecasting," *IEEE Power and Energy Society General Meeting – Conversion and Delivery of Electrical Energy in the 21st Century*, pp. 1-6, 2008.

[18] E.A.Wan, R. Van Der Merwe, "The unscented Kalman filter for nonlinear estimation", in Adaptive Systems for Signal Processing, Communications, and Control Symposium 2000, pp. 153-158, 2000.

[19] Z. Jian, A. Swain, N. K. C. Nair, and J. J. Liu, "Estimation of power quality using an unscented Kalman filter," TENCON 2007 - 2007 IEEE Region 10 Conference, 2007, pp. 1-4.

[20] P.K. Dash, S. Hasan and B.K. Panigrahi, "Adaptive complex unscented Kalman filter for frequency estimation of time-varying signals," *IET Science, Measurement and Technology*, vol. 4, no. 2, pp. 93-103, 2010.

[21] J.B.V. Reddy, P.K. Dash, R. Samantaray and K. Moharana, "Fast tracking of power quality disturbance signals using an optimized unscented filter," *IEEE Transactions on Instrumentation and Measurement*, vol. 58, no. 12, pp. 3943-3952, 2009.

[22] A.E.Emanuel, "Summary of IEEE Standard 1459: Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions", IEEE Transactions On Industry Applications, Vol. 40, No. 3, May/June 2004, pp.869-876.

VII. BIOGRAPHIES

Happy Novanda obtained the B.Sc degree in Electrical Engineering from the University of Indonesia in 2006 and the Master degree in Technology Management from the University of New South Wales in 2007. Currently she is a PhD student at the University of Manchester working in the area of development of estimation methods for estimation of power quality indices and investigating the nature of harmonic distortions at renewable energy resources, particularly inverter connected wind farms.

Pawel Regulski obtained the MSc degree in Electrical Engineering from the Wroclaw University of Technology (Poland) in 2008. Currently he is a PhD student at the University of Manchester working on load modeling and voltage stability. His main research interests include development of new estimation techniques and implementation of artificial intelligence techniques in power system applications.

Francisco M. González-Longatt (S'01, M'03, SM' 2009) was born in Cagua-Venezuela, on July 30, 1972. He graduated on Electrical Engineering of Instituto Universitario Politécnico de la Fuerza Armada Nacional, Venezuela (1994), and Master of Business Administration of Universidad Bicentenaria de Aragua, Venezuela (1999) and PhD of Universidad Central de Venezuela (2008). His main area of interest is renewable energy resources and power system dynamic in smart grids. He is former associate professor on Electrical engineering Department of Universidad Nacional Politécnico de la Fuerza Armada Nacional, Venezuela. He is currently a Post Doctoral Associate Research School of Electrical and Electronic Engineering, The University of Manchester.

Vladimir Terzija (M'95, SM'2000) is the EPSRC Chair Professor in Power System Engineering in the School of Electrical and Electronic Engineering, The University of Manchester, where he has been since 2006. From 1997 to 1999, he was an Assistant Professor at the University of Belgrade. In 1999, he was awarded a prestigious Humboldt Research Fellowship. From 2000 to 2006, he was with ABB AG, Germany, working as an expert for switchgear and distribution automation. His main research interests are application of intelligent methods to power system monitoring, control, and protection, switchgear and fast transient processes, as well as DSP applications in power systems.